

# **Modelling the Arrival Process for Packet Audio**

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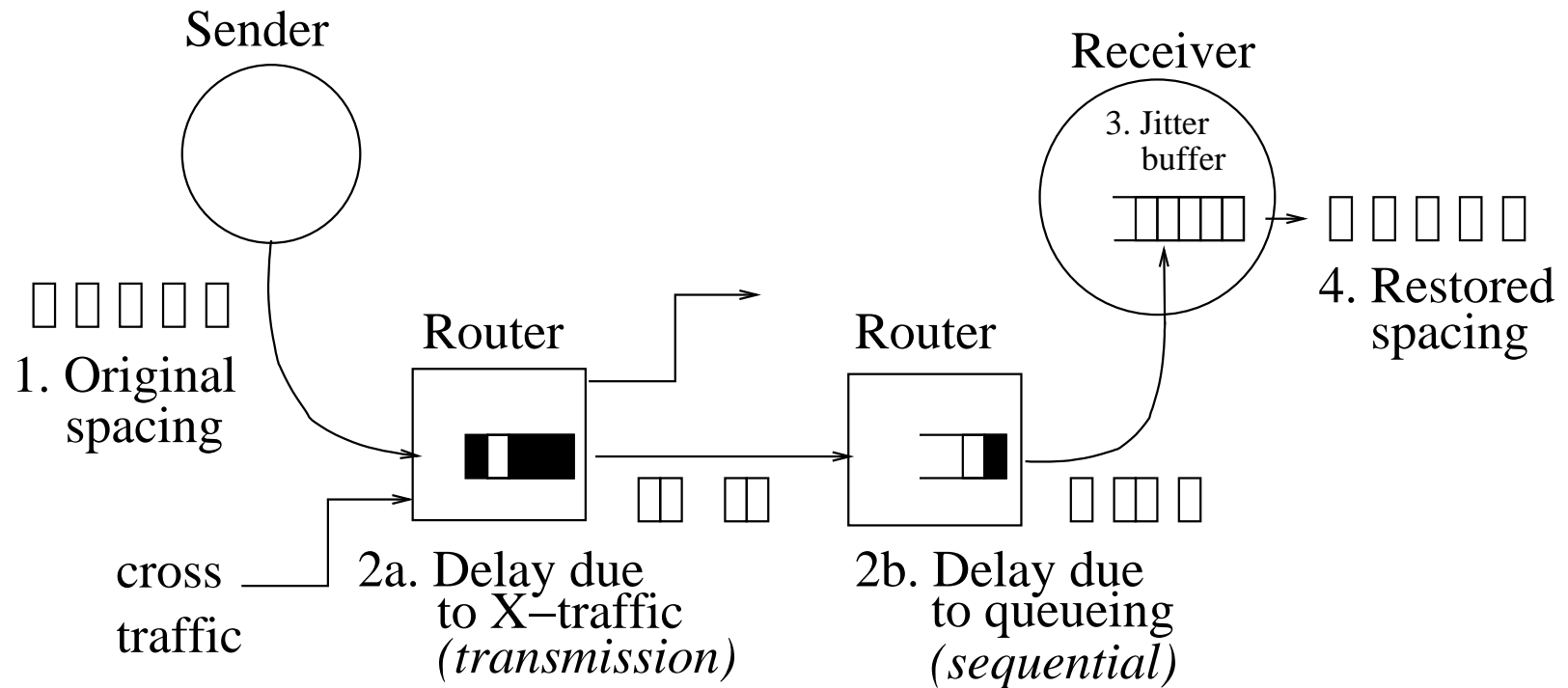
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## **Mixed Model and Measurement Approach**

- Models are often flexible but they are hard to map to real systems, little room for inclusion of real data which is often approximated e.g the choice of arbitrary packet loss values
- Measurements are representative but often inflexible e.g. Would like to investigate the effect of packet size, which means often means re-running the experiments, proposed solution - incorporate real data

# Effect of Network on Packet Audio Spacing



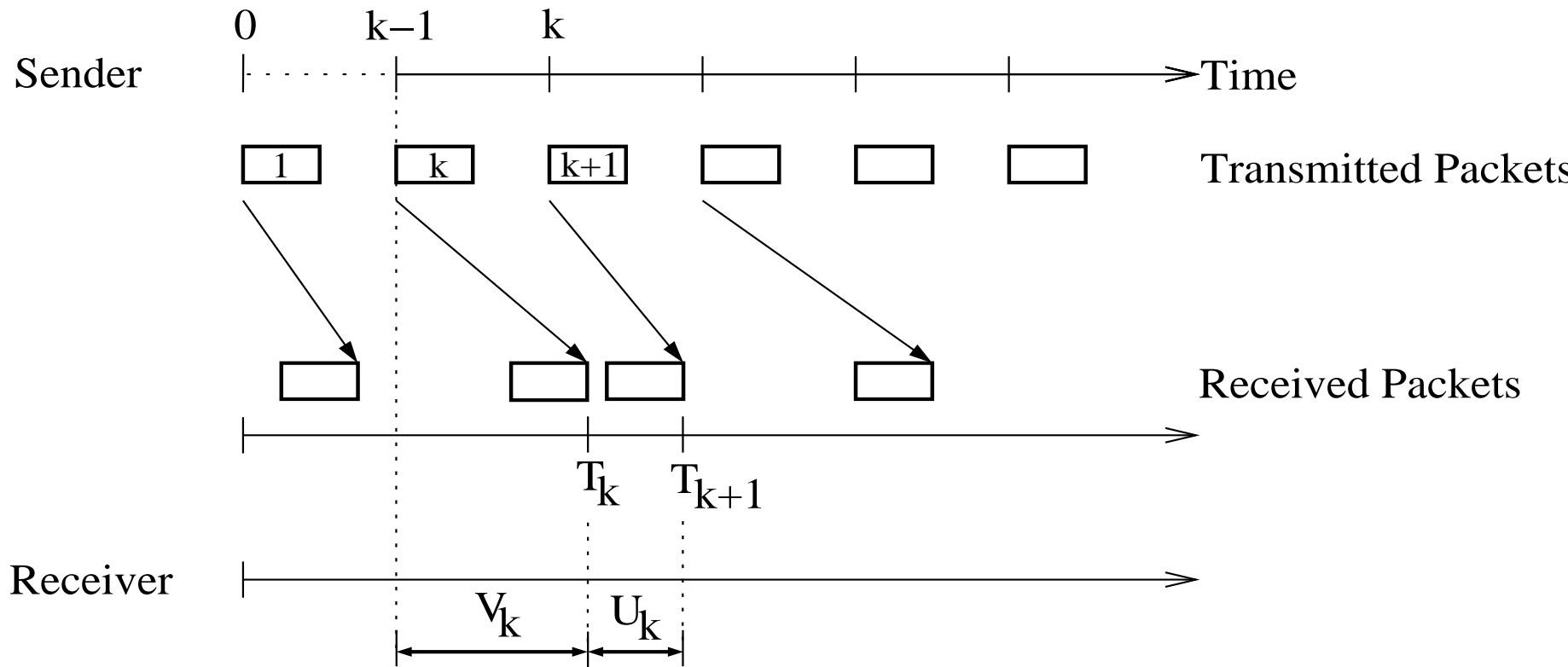
Steps 1 to 4 show the distortion/restoration of packet spacing

Transmission delay from cross-traffic & sequential delay due to the same stream

## Model Preparation

- VoIP data is sent with constant packet spacing
- In this case 20ms (160 byte payloads)
- We want to investigate the variation in the network
- We use time and packet number interchangeably, sort of *duality*

# Packet Delay Model



$k$  are packets,  $T_k$  arrival times,  $V_k$  observed delays,  $U_k$  observed interarrival times

## Model Variables

*Arrival times ( $T_k$ )*

*Observed delays ( $V_k$ ):*

- arrival time at receiver - departure time from sender =  $T_k - k + 1$ ,

*Observed interarrival times ( $U_k$ ):*

- time between consecutive arrivals at receiver  $U_k = T_{k+1} - T_k$

*Transmission delays ( $Y_k$ ):*

General unknown distribution, we assume iid with a

distribution function:  $F(x) = P(Y_k \leq x)$ ,  $k \geq 1$

mean  $\nu = \int_0^\infty (1 - F(x)) dx < \infty$  (with typical values 20-40)

## Markovity in the Model

Packet  $k$  needs  $Y_k$  to propagate through network

If it is not further delayed then it arrives at  $T_k = k - 1 + Y_k$

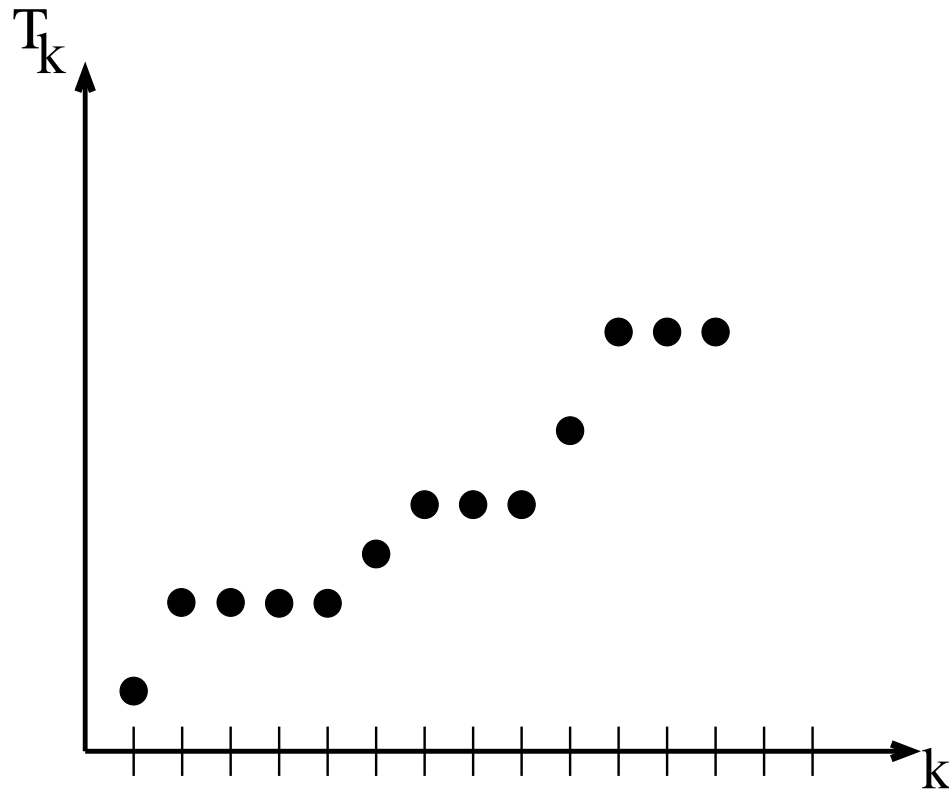
*But* if delayed by packets in front of it (sequential delay),  
 $1 \rightarrow (k - 1)$  then:

$$T_k = \max(T_{k-1}, k - 1 + Y_k), \quad k \geq 2 \quad (T_1 = Y_1)$$

Arrival times ( $T_k$ ) form a transient Markov chain, 2 conditions needed for Markovity :

- Probability only depends on present state (memoryless)
- Probabilities don't change at each step (time homogen.)

## Inter-arrival times II



Interarrival time  $T_k$  shown against packet number  $k$

$T_k$  increases with time

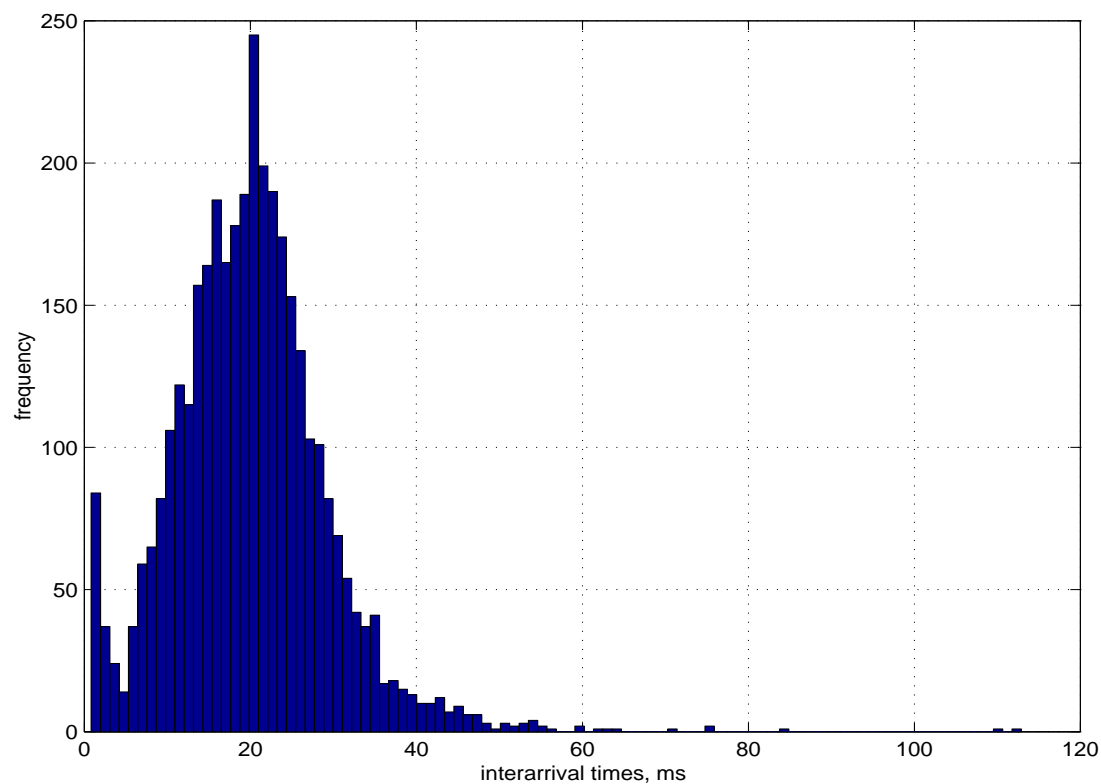
## Inter-arrival times

The inter-arrival times satisfy:

$$U_k = T_k - T_{k-1} = \max(0, k - 1 + Y_k - T_{k-1}) \quad k \geq 2.$$

Note:  $T_k$  and  $U_k$  can be observed from traffic traces.

# Histogram of observed inter-arrival times ( $U_k$ )



VoIP session from Argentina to Stockholm, packetisation 20ms

## Expected Value of $U_k$

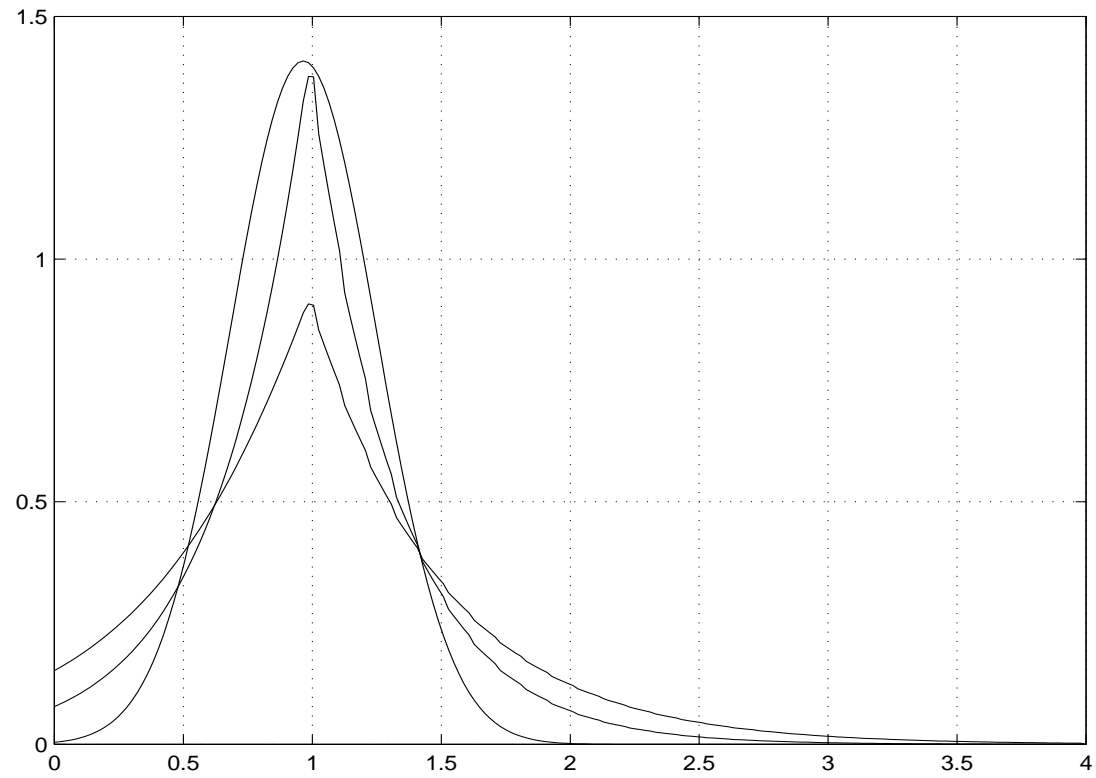
In the expression

$$E(U_k) = 1 + \int_1^\infty P(Y_1 > t)P(T_{k-1} \leq t - 1) dt \rightarrow 1,$$

$t$  is a fixed quantity, but eventually  $T_k$  will be greater than  $t$ , however  $P(Y_1 > t)$  is finite. Can use the dominated convergence theorem to show the whole integral will disappear.

Next slide shows numeric approximations of the (non-normalised) density function  $\frac{d}{dx}P(U_\infty \leq x)$  for three choices of  $F$ .

# Interarrival Density of $(U)$



## U density Plot Explained

3 choices of an arbitrary function  $F$ , all have a peak  $E_\infty \approx 1$

The peak at 1 is akin to the majority of packets arriving at the packetisation interval ( $E(U_\infty) \approx 1$  which is 20ms)

If one fixes  $x = 0$  i.e.  $P(U_\infty = 0)$  this is in correspondence with a number of packets arriving back to back

Highest peak is Gaussian(5, 2) (Mean, Variance). The lowest peak is an exponential (Variance = 3). In between is also exponential distribution with variance = 2

# Silence Suppression I

Silence suppression is an additional source of random delay  
Introduced to reduce load on network when talker is silent

$X_k$  = duration of silence between packets  $k - 1$  and  $k$ .

We assume that the silence suppression intervals are  
independent of  $(Y_k)_{k \geq 1}$

$$G(x) = P(X_k \leq x),$$

$$S_k = \sum_{i=1}^k X_i = \text{total time of silence suppression affecting packet } k$$

## Silence Suppression II

which implies that the delivery of packet  $k$  from the sending unit now starts at time  $k - 1 + S_k$ . The representation takes the form

$$T_1 = S_1 + Y_1, \quad T_k = \max(T_{k-1}, k - 1 + S_k + Y_k), \quad k \geq 2,$$

hence the inter-arrival times are modified so:

$$U_k = T_k - T_{k-1} = \max(0, k - 1 + S_k + Y_k - T_{k-1}) \quad k \geq 2.$$

In the paper proof that :

$$E(U_k) \rightarrow 1 + \mu, \quad E(V_k) \rightarrow \nu + \int_1^\infty P(Y_1 > t) E(N(t-1)) dt,$$

## Packet Loss I

Without silence suppression, probability  $p$  that a packet is subject to loss, independent of other losses and transmission delays.

$K_k$  = number of attempts required between successful packets  $k - 1$  and  $k$ ,  $k \geq 1$

Which gives a sequence  $(K_k)_{k \geq 1}$  of independent, identically distributed random variables with the geometric distribution

$$P(K_k = j) = (1 - p)p^j, \quad j \geq 0.$$

## Packet Loss II

$L_k = K_1 + \dots + K_k$  no. of attempts required for  $k$  successful packet

$L_k$  is a sequence of random variables with a negative binomial distribution. The arrival times are now given by

$$T_1 = K_1 - 1 + Y_{K_1}, \quad T_k = \max(T_{k-1}, L_k - 1 + Y_{L_k}), \quad k \geq 2.$$

Again in the paper there is a proof:

$$T_k = K_1 - 1 + \max(Y_1, 1 + T_{k-1}), \quad k \geq 2$$

## Model Results

Combining loss and silence suppression:

$$E(U_k) \rightarrow 1 + E(X) + E(K_1 - 1) = \mu + \frac{1}{1-p}, \quad k \rightarrow \infty,$$

with  $K_1$ ,  $Y_1$  and  $T_{k-1}$  all independent.

This is the same relation as before with  $X_1$  replaced by  $K_1 - 1$  and hence, as before  $E(U_k) \rightarrow 1 + E(K_1 - 1) = \frac{1}{1-p}$ ,  $k \rightarrow \infty$ , which provides a simple method to estimate packet loss based on observed interarrival times.

## Numerical Estimate of $U$ and Loss Probability

Can estimate  $u_k$  (average inter-arrival time) from trace data e.g.

$$u = \frac{1}{n} \sum_{k=1}^n u_k \text{ ms.}$$

With no silence suppression, given an observed realization  $(u_k)_{k=1}^n$  of  $(U_k)$ , a point estimate of the packet loss probability  $p$  is obtained (with silence interval  $\mu = 0$ ), using

$$p^* = 1 - \frac{20}{\bar{u}},$$

Our measurements gave consistently  $\bar{u} \approx 20.002 - 20.005$  ms, indicating loss probabilities of the order  $10^{-4}$ .

## Estimating the Transmission Delay $Y$

Given a fixed length sample observation  $(v_k)$  of the Markov chain  $(V_k)$  for observed delays. From the simple relation:

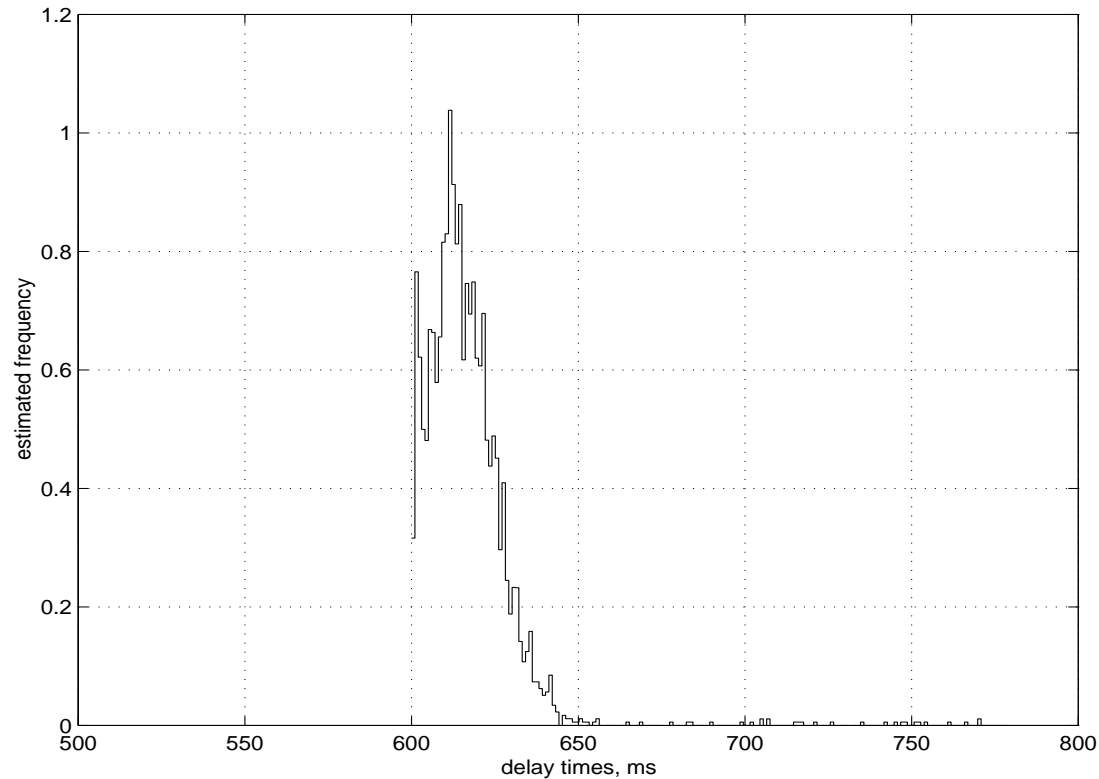
$$P(V_\infty \leq x) = F(x) \prod_{i=1}^{\infty} F(x+i) = F(x) P(V_\infty \leq x+1)$$

shows that if we let  $\bar{F}_V$  denote an empirical distribution function of  $V$ , then we obtain an estimate  $\bar{F}$  of  $F$  by taking

$$\bar{F}(x) = \frac{\bar{F}_V(x)}{\bar{F}_V(x+1)} \quad x \geq 0,$$

Yields:

# Estimated Density Function of $Y$



Similar to the observed delays, it is sensitive for small changes in the data (shift to smaller values of  $Y$  compared to  $V$ ).

## Conclusions

- Developed an extensible model for VoIP arrivals
- Includes silence suppression and loss
- Relates inter-arrival times to loss
- Some insight into the cause of delay  
(sequential/transmission) delays
- Can make a traffic generator from the distributions of  $U_k$

## References

*Paper:* [http://www.sics.se/~ianm/Papers/audio\\_delay\\_sub\\_cr.pdf](http://www.sics.se/~ianm/Papers/audio_delay_sub_cr.pdf)

*Presentation:* [http://www.sics.se/~ianm/Talks/audio\\_delay.pdf](http://www.sics.se/~ianm/Talks/audio_delay.pdf)

*Book:* Ingemar Kaj - SIAM Monograph on Mathematical Modeling and Computation: Stochastic Modeling in Broadband Communications Systems (pp 96-100)