Enterprise Architecture Analysis with Production Functions

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EA analysis
Consider a firm that produces an output good $y$ using factor inputs $x_1, x_2, \ldots x_n$. The output might be cars, say, and the factor inputs might be labor, capital and raw materials. The production function $f(x_1, x_2, \ldots x_n) = y \in \mathbb{R}$ such that $y$ is the maximum output associated with spending $x_1, x_2, \ldots x_n$. 
One output good from two inputs
An optimization problem

A cost-minimizing firm producing $y$ has to solve the following constrained optimization problem:

$$\min_x \sum_{i=1}^{n} w_i x_i$$

such that $f(x_1, x_2, \ldots, x_n) = y$

where $w_1, w_2, \ldots, w_n$ are the prices of factor inputs $x_1, x_2, \ldots, x_n$, respectively.
First-order optimality

The $n$ first order conditions for an optimal solution to this problem can be found using Lagrange multipliers. Dividing the $i$:th and $j$:th conditions yields the following condition, where we denote an interior solution point $(x_1^*, x_2^*, \ldots, x_n^*) \in \mathbb{R}^n$ by the vector $\mathbf{x}^*$:

$$\frac{w_i}{w_j} = \frac{\partial f(\mathbf{x}^*)}{\partial x_i} \frac{\partial f(\mathbf{x}^*)}{\partial x_j}$$

Economic rate of substitution (ERS)  \hspace{1cm}  Technical rate of substitution (TRS)  

(2)
The impact of prices
Why is this useful?

- Production economics can answer questions how good different enterprise architectures are – if there is a good theory to build on!
- Three examples:
  - IT productivity and growth strategies for a company
  - Strategies for high availability IT services
  - Composition of a military unit
Example 1: IT and productivity

In a widely cited paper, Hitt and Brynjolfsson estimate output elasticities in a Cobb-Douglas model for the productivity of 370 firms [3]. Three factor inputs are used: total IT stock $C$, non-computer capital $K$ and labor $L$, yielding the following Cobb-Douglas model for value added $V$ (econometric dummy variables removed):

$$V = C^{\beta_1} K^{\beta_2} L^{\beta_3}$$

(4)

**TABLE I.** Output elasticities for total IT stock, non-computer capital and labor, from [3].

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0883</td>
<td>0.212</td>
<td>0.663</td>
</tr>
</tbody>
</table>

Extensive or intensive growth?

As-is

To-be?
The two growth strategies are basically about re-adjusting the production factor of IT-stock $C$ and labor $L$. Applying the conditions expressed in Eq. (2) to the model for $V$ specified by Hitt and Brynjolfsson in Eq. (4), we have:

$$\frac{\frac{\partial V}{\partial C}}{\frac{\partial V}{\partial L}} = \beta_1 C^{\beta_1 - 1} K^{\beta_2} L^{\beta_3} = \frac{\beta_1}{\beta_3} \frac{L}{C}$$

(6)

**TABLE II.** A NUMERICAL COMPARISON OF THE THREE SCENARIOS FOR THE FINANCIAL ANALYSIS FIRM. ALL COSTS ARE PER ANNUM, AND INTENDED TO BE BALLPARK-REALISTIC IN $ or €. FOR THE IT DEVICES, THIS SHOULD BE A DEPRECIATED PRICE, E.G. THE LIST PRICE DIVIDED BY THE NUMBER OF YEARS IN USE.

<table>
<thead>
<tr>
<th></th>
<th>As-is</th>
<th>Extensive to-be</th>
<th>Intensive to-be</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>300</td>
<td>400</td>
<td>250</td>
</tr>
<tr>
<td>Devices</td>
<td>330</td>
<td>430</td>
<td>330</td>
</tr>
<tr>
<td>IT Labor</td>
<td>13</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>IT Stock</td>
<td>369</td>
<td>475</td>
<td>360</td>
</tr>
<tr>
<td>LaborCost</td>
<td>37000</td>
<td>37000</td>
<td>45000</td>
</tr>
<tr>
<td>DeviceCost</td>
<td>400</td>
<td>250</td>
<td>300</td>
</tr>
<tr>
<td>IT LaborCost</td>
<td>40000</td>
<td>38000</td>
<td>45000</td>
</tr>
<tr>
<td>IT StockCost</td>
<td>4585</td>
<td>3826</td>
<td>4025</td>
</tr>
<tr>
<td>TRS</td>
<td>0.108</td>
<td>0.112</td>
<td>0.092</td>
</tr>
<tr>
<td>ERS</td>
<td>0.124</td>
<td>0.103</td>
<td>0.089</td>
</tr>
<tr>
<td>TRS/ERS</td>
<td>0.87</td>
<td>1.08</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Example 2: High availability strategies

• Consider an e-commerce firm, where the availability of the electronic sales application service is essential
  – This is where revenue is made, and where customers are lost

• Business operations impose a demand for an availability level of 99.81%
  – ”Continuous availability” according to Gartner

• How should the CIO meet this requirement?
A simple mathematical model

Steady state availability (roughly mean uptime over a time interval) is defined mathematically as follows:

$$A = \frac{MTTF}{MTTF + MTTR}$$  \hspace{1cm} (7)

MTTF denotes "Mean Time To Failure" and MTTR "Mean Time To Repair" or "Mean Time To Restore". As mean times are used, Eq. (7) measures the long-term performance of a system. Now, if we consider availability the output good,
High availability, cont’d

Two possible strategies to increase $A$:

- Increase MTTF (the time until a failure occurs)
- Decrease MTTR (the time to repair a failure)
Increase MTTF

With "more" capital K you may for instance:

- By better hardware that lasts longer
- Build redundant systems with fail-over
- Install continuous monitoring of your service and perform just-in-time preventive maintenance rather than expensive corrective maintenance

\[ A = \frac{MTTF}{MTTF + MTTR} \]

\[ MTTF = k_2 K^{\beta_2} \]

\( \beta_2 \) may be as great as 1, but hardly all the time
Decrease MTTR

With "more" labor $L$ you may for instance:

- Hire more people to the repair crew, who can work in parallel (or repair difficult problems together)
- Hire more qualified people to the repair crew, who can repair failures quicker (or repair difficult problems on their own)

$$MTTR = k_3 L^{-\beta_3}$$

$\beta_3$ may be as great as 1, but hardly all the time
Optimal availability investment

Assuming linear costs $w_L$ och $w_K$ for L and K:

$$\min_{L,K} w_L L + w_K K$$

such that

$$\frac{k_2 K^{\beta_2}}{k_2 K^{\beta_2} + k_3 L^{1-\beta_3}} = A^*$$

Optimal solution to reach $A^*$:

$$L^*(w_L, w_K, A^*) = \left( \frac{w_L \beta_2}{w_K \beta_3} \right)^{-\frac{\beta_2}{\beta_2 + \beta_3}} \left( \frac{A^*}{k_3} \frac{k_3}{1 - A^* k_2} \right)^{\frac{1}{\beta_2 + \beta_3}}$$

$$K^*(w_L, w_K, A^*) = \left( \frac{w_L \beta_2}{w_K \beta_3} \right)^{-\frac{\beta_3}{\beta_2 + \beta_3}} \left( \frac{A^*}{k_3} \frac{k_3}{1 - A^* k_2} \right)^{\frac{1}{\beta_2 + \beta_3}}$$
Optimal architecture example

Assuming:
\[ k_3 = 200, \beta_3 = 0.663, \]
\[ k_2 = 20000, \beta_2 = 0.212, \]
\[ w_L = 40000, w_K = 300 \]

Yields:
\[ K^* = 115 \]
\[ L^* = 3 \]
Example 3: Composition of a military unit

Example organizations from SNDC
A simple mathematical model

The Lanchester equations for attrition combat (Hildebrandt, 1999):

\[
\frac{dB}{dt} = -Q_R B^s R
\]
\[
\frac{dR}{dt} = -Q_B R^s B
\]

The outcome is determined by the quality and quantity of the belligerents. 0 ≤ s ≤ 2. For s = 0 spelar only quantity matters, for s = 2 only quality.
The Lanchester equations yield an attrition quotient $\phi$ between red and blue (Hildebrandt, 1999):

$$\phi = \frac{Q_B B^{2-s}}{Q_R R^{2-s}}$$

We set $s \approx 0.5$ (Osipov, 1915), use standard weights to form weighted quantities $B$ and $R$ for soldiers, APCs, tanks and artillery (Bracken, 1995), and make assumptions about the relative qualities $Q_B$ och $Q_R$. 
Numerical evaluation

<table>
<thead>
<tr>
<th>Brigade</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.64</td>
<td>1.61</td>
<td>1.46</td>
</tr>
</tbody>
</table>

Bar chart showing the values for each brigade.
Different model maturities

- The growth strategies case uses mature research – 370 firms econometrically analyzed by Hitt and Brynjolfsson.
- In the availability case, assumptions have to be made about production functions, manpower shortage, statistical distributions.
- In the military unit case, we use an established model – but one that is known to be a radical simplification.
Two kinds of alternatives

• Use-case where a proposed technical solution already on the table is evaluated from the point of view of economic efficiency

• Use-case where an optimal technical solution is generated from the microeconomics
  – Remember that production functions are black-box abstract models of technology – not blueprints for particular solutions.
Conclusions

• Production economics can answer questions how good different enterprise architectures are.
• The production economics approach can be integrated into an EA analysis tool.
• A decision-maker can get advanced decision-support without having to master production economics.